

# Basic Algebra Helper

## ANSWER KEY

### PART 1

The goal of *algebra* is to find the value of an unknown quantity, which is often represented by a letter, such as "x" or "y". The solution is written, "x = \_\_\_\_". If the *variable* (unknown quantity, unlike a *constant*, such as the number 2) is "x", then you must "solve the *equation* for x". An *equation*, such as  $2x+1=9$ , has two sides, which are *equated* (said to be *equal*) to each other. The two sides must balance out. With the correct value *substituted* for "x", the two sides balance out, and the equation is solved. In fact, basic arithmetic is like algebra! " $3 + 5 = \_\_\_$ " is really  $3+5=x$ . What is the value of "x"?

### PART 1 EXERCISES

## ANSWERS

\* Solve the following equations for "x".

1) $3+2=x$ $x=5$	2) $14-6=x$ $x=8$	3) $7\times 5=x$ $x=35$
4) $18\div 3=x$ $x=6$	5) $2\times 8=x$ $x=16$	

### PART 2

Do you remember math problems, like " $5 + \_\_\_ = 10$ "? That equation is really  $5+x=10$ . How do you find the value of "x"? You need to get "x" on one side of the equation, and a number on the other side. If you subtract 5 from one side, you must subtract 5 from the other side.

$5+x-5=10-5$ . This can be simplified to  $x=5$ , which is the answer. You can add, subtract, multiply, or divide both sides by any quantity (variable or constant). However, you must always remember to manipulate both sides in the exact same way!

## PART 2 EXERCISES

### ANSWERS

\* Solve the following equations for "x".

6) $7 - x = 4$ $7 - x + x = 4 + x$ $7 - 4 = 4 + x - 4$ $3 = x$ $x = 3$	7) $x \times 6 = 24$ $x \times 6 \div 6 = 24 \div 6$ $x = 4$	8) $9 + x = 10$ $9 + x - 9 = 10 - 9$ $x = 1$
9) $30 \div x = 6$ $30 \div x \times x = 6 \times x$ $30 \div 6 = 6 \times x \div 6$ $5 = x$ $x = 5$	10) $x + 4 = 13$ $x + 4 - 4 = 13 - 4$ $x = 9$	

## PART 3

Sometimes, it takes several steps to get the equation in the form, " $x = \underline{\quad}$ ". For example,  $2x + 1 = 9$ .  $2x$  means "2 times x". **Whatever you do to one side of the equation, you must do to the other side.** Subtract 1 from both sides:  $2x + 1 - 1 = 9 - 1$  simplifies to  $2x = 8$ . We now know that 2 x's make 8, but what does one "x" make? Divide both sides by 2 to find out:  $\frac{2x}{2} = \frac{8}{2}$ , which simplifies to  $x = 4$ .

## PART 3 EXERCISES

### ANSWERS

\* Solve the following equations for "x".

11) $4x + 1 = 17$ $4x + 1 - 1 = 17 - 1$ $\frac{4x}{4} = \frac{16}{4}$ $x = 4$	12) $15 - 2x = 7$ $15 - 2x - 15 = 7 - 15$ $\frac{-2x}{-2} = \frac{-8}{-2}$ $x = 4$	13) $9x + 9 = 27$ $9x + 9 - 9 = 27 - 9$ $\frac{9x}{9} = \frac{18}{9}$ $x = 2$
14) $14 - 3x = 5$ $14 - 3x - 14 = 5 - 14$ $\frac{-3x}{-3} = \frac{-9}{-3}$ $x = 3$	15) $5x + 10 = 35$ $5x + 10 - 10 = 35 - 10$ $\frac{5x}{5} = \frac{25}{5}$ $x = 5$	

## PART 4

What do you do if you have constants and variables on both sides of the equation? Example:

$5x - 6 = 3x + 12$ . You must get all of the variables over to one side, and you must get all of the constants over to the other side. **Whatever you do to one side of the equation, you must do to the other side.** Add 6 to both sides:  $5x - 6 + 6 = 3x + 12 + 6$  simplifies to  $5x = 3x + 18$ . Subtract  $3x$  from both sides:  $5x - 3x = 3x + 18 - 3x$  simplifies to  $2x = 18$ . Divide both sides by 2:  $\frac{2x}{2} = \frac{18}{2}$ , which simplifies to  $x = 9$ . **You can flip the sides:**  $x = 9$  **is the same as**  $9 = x$ . That is the equivalent of subtracting 9 from both sides:  $x - 9 = 9 - 9$  ( $x - 9 = 0$ ), subtracting "x" from both sides:  $x - 9 - x = 0 - x$  ( $-9 = -x$ ), and then multiplying each side by  $-1$ :  $-1(-9) = -1(-x)$  ( $9 = x$ ). **Negation is like multiplying by  $-1$ .**

## PART 4 EXERCISES

### ANSWERS

\* Solve the following equations for "x".

<p>16) <math>3x + 4 = 8 + x</math>  <math>3x + 4 - 4 = 8 + x - 4</math>  <math>3x - x = 4 + x - x</math>  <math>\frac{2x}{2} = \frac{4}{2}</math>  <math>x = 2</math></p>	<p>17) <math>6x - 12 = 5x - 12</math>  <math>6x - 12 + 12 = 5x - 12 + 12</math>  <math>6x - 5x = 5x - 5x</math>  <math>x = 0</math></p>	<p>18) <math>3x - 8 = 2x + 1</math>  <math>3x - 8 + 8 = 2x + 1 + 8</math>  <math>3x - 2x = 2x + 9 - 2x</math>  <math>x = 9</math></p>
<p>19) <math>9x - 1 = x - 9</math>  <math>9x - 1 + 1 = x - 9 + 1</math>  <math>9x - x = x - 8 - x</math>  <math>\frac{8x}{8} = \frac{-8}{8}</math>  <math>x = -1</math></p>	<p>20) <math>3 - x = 2x + 9</math>  <math>3 - x - 3 = 2x + 9 - 3</math>  <math>-x - 2x = 2x + 6 - 2x</math>  <math>\frac{-3x}{-3} = \frac{6}{-3}</math>  <math>x = -2</math></p>	

## PART 5

There are other operations that can be done to both sides of an equation, such as exponentiation. Example:  $x^2=25$  . If "x" squared is 25, then what is "x"? Take the square root of both sides (the equivalent of raising both sides to the 1/2th power):  $\sqrt{x^2}=\sqrt{25}$  , and you get  $x=5$  . Actually, "x" can also equal  $-5$ , because  $-5$ , multiplied by itself, also equals 25! Let's try another equation:  $\sqrt{x}=4$  . Square both sides:  $(\sqrt{x})^2=4^2$  , and you end up with  $x=16$  . If "x" is raised to the Nth power, the *inverse* operation ("the opposite"; for example, subtraction is the inverse of addition) is to take the Nth root of it (which is the equivalent of raising it to the 1/Nth power). For example, take the cube root of  $x^3$  to get "x". If the Nth root of "x" is taken, the inverse operation is to raise it to the Nth power. For example, raise  $\sqrt[4]{x}$  to the fourth power to get "x".

Finally, division by 0 is impossible, and on this level, you cannot take an *even* (square, fourth, sixth, ...) root of a negative number. Also, remember the order of operations, "PEMDAS" (Parentheses, Exponents, Multiplication, Division, Addition, Subtraction). Some people remember this acronym by the *mnemonic*, "Please Excuse My Dear Aunt Sally". For a *term* (a value that either exists by itself or is separated from other terms by plus or minus signs), like  $2x^2$  , 2 is multiplied by the value of "x" squared. This is NOT equivalent to  $(2x)^2$  , where 2 is multiplied by the value of "x", and then THAT value is squared.  $(2x)^2=4x^2$  !

## PART 5 EXERCISES

### ANSWERS

\* Solve the following equations for "x".

21) $x^2=100$ $\sqrt{x^2}=\sqrt{100}$ $x=10, -10$	22) $4=\sqrt{x}$ $4^2=(\sqrt{x})^2$ $16=x$ $x=16$	23) $x^3-1=7$ $x^3-1+1=7+1$ $\sqrt[3]{x^3}=\sqrt[3]{8}$ $x=2$
24) $2x^2+1=33$ $2x^2+1-1=33-1$ $\frac{2x^2}{2}=\frac{32}{2}$ $\sqrt{x^2}=\sqrt{16}$ $x=4, -4$	25) $5x^2-1=x^2+15$ $5x^2-1+1=x^2+15+1$ $5x^2-x^2=x^2+16-x^2$ $\frac{4x^2}{4}=\frac{16}{4}$ $\sqrt{x^2}=\sqrt{4}$ $x=2, -2$	